AUTOMATED INTELLIGENT MODE SELECTION FOR FAST MODE MATCHING ANALYSIS OF WAVEGUIDE DISCONTINUITIES

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Abstract **-- This paper presents a novel scheme for mode selection in mode matching problems. The technique is capable of reducing the size of a system considerably whilst retaining high accuracy. A procedure for efficient cascade analysis with the technique is also presented.**

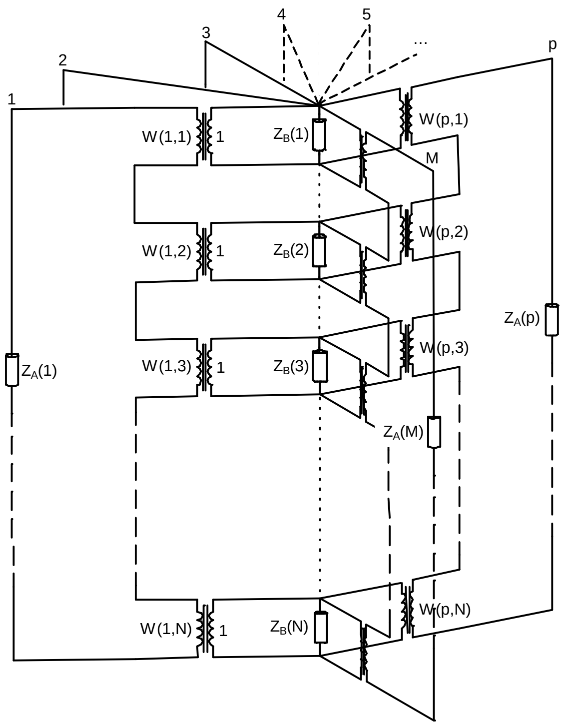

Fig 1. A mode matching equivalent circuit with all ports terminated by their characteristic impedances.

I. INTRODUCTION

The mode matching technique was first proposed by Wexler in 1967 [1]. The usefulness and accuracy of his method assured it a great deal of attention from researchers since that time [2]-[4]. In its present form, mode matching is used extensively for the analysis of many types of waveguide discontinuity problems and promises an exciting future through its potential hybridization with other codes.

Mode matching entails the expansion of tangential fields on both sides of a discontinuity into infinite summations of modes with unknown complex amplitudes. These amplitudes are then calculated by enforcing the boundary conditions at the discontinuity plane. In order to render the technique numerically viable, only a finite number of modes in each guide must be considered. This creates a convergence issue that has been discussed in the literature [5]. The essence of this issue is that enough modes must be used to ensure convergence. For discontinuities that differ markedly from the embedding structure, this number can be very high.

The typical solution to this problem is only valid for cases where the step geometry is very simple, or excitation is only by one simple mode, e.g. TE_{10}. In this case the problem is expressed in terms of alternate mode sets, e.g. TE-to-x/TM-to-x, where symmetry principles are used to argue that one type of mode, e.g. TM-to-x, will not be excited by the step at all, and can be safely neglected in the mode matching operation. While this approach does result in more reasonable sized problems, it does have certain drawbacks.

First, it cannot be used with a great deal of consistency in cascaded systems of steps that do not all adhere to the symmetry conditions. Second, the calculated results of such an analysis are in terms of the alternate mode set, requiring conversion to the more standard TE-to-z/TM-to-z formulation [6]. Third, the great flexibility of the mode matching method is compromised by the limitations on step geometry and modal excitation imposed by the approach. Finally it does not lend itself well to generalization; each problem must be separately considered by the user and an applicable code written for it.

This paper proposes a new method of intelligent mode selection based on an improved equivalent circuit representation of the mode matching equations. The technique can be used to systematically reduce the mode matching content to contain only the critical interacting modes for a particular problem. We also show how this technique can be used to solve problems of cascaded steps with significant improvements in speed over the conventional approach. Finally, excellent physical insight

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can be gained in an easy manner. The approach does not rely on the use of alternate mode sets and does not limit the modal excitation. It can also be used for discontinuities of any nature (e.g. rectangular-to-circular guide).

The paper will start by showing the mode matching equations as derived in the literature. It will then give the equivalent circuit that represents these equations and show how it can be used to identify the modes that can be neglected. The technique will then be applied to a step discontinuity in rectangular waveguide and compared with the standard technique. The second part of the paper will show how the approach can be used to solve large cascaded systems.

II. Circuit Representation

The mode matching equations, derived in a similar fashion as in [4], for a step discontinuity are as in (1).

$$\begin{aligned} &\overline{V}_A = [W]\overline{V}_B \quad \text{with } \overline{V}_x = diag\left(\sqrt{\overline{Z}_x}\right)\cdot\left(\overline{a}_x + \overline{b}_x\right) \\ &\overline{I}_B = -[W]^T\,\overline{I}_A \quad \text{with } \overline{I}_x = diag\left(\sqrt{\overline{Y}_x}\right)\cdot\left(\overline{a}_x - \overline{b}_x\right) \\ &\text{with} \quad x \in (A,B) \quad \text{and } S_A > S_B \\ &\text{and} \quad W(m,n) = \iint_{S_B} (\overline{e}_n^B \times \overline{h}_m^A)\cdot d\hat{z} \end{aligned} \qquad (1)$$

Here, $\overline{a}$ and $\overline{b}$ are the normalized complex mode amplitudes at ports *A* and *B*. $\overline{Z}$ and $\overline{Y}$ are the respective modal wave impedances and admittances which define $\overline{V}$ and $\overline{I}$, the equivalent voltages and currents.

The entries of the [*W*] matrix are calculated from the double integral over the common aperture of the step, S_B, of the cross product of the normalized frequency independent modal field patterns on either side of the step. This matrix is frequency independent.

These equations can be represented by an equivalent circuit network shown in Fig. 1. This circuit is similar to that proposed by [7] except that their circuit is used exclusively for H-plane steps. In addition, we arrive at this circuit directly from the mode matching equations, making it applicable to all mode matching problems that can be expressed in the form of (1). In our view, this representation of the equivalent circuit is clearer and gives excellent physical insight. For the sake of simplicity the circuit is drawn with all ports terminated by their characteristic (wave) impedances. These ports would, in practice, be extended as transmission lines (as illustrated in [7]) to the next discontinuity or physical port.

III. Subcircuit Reduction

Consider now the case of an ideal transformer coupling mode *m* on side A with mode *n* on side B. In this case the transformer turns ratio is *W(m,n)*. If this value becomes very low, the low voltage side of the ideal transformer occurs in series on the A side and the low current side occurs in shunt on the B side. As *W(m,n)* is reduced to zero, the transformer will look increasingly like a short circuit from the A side and an open circuit from the B side. In the limit where *W(m,n)* tends to zero, the transformer can be completely neglected, or removed from the circuit, cutting the connection between modes *m* and *n*.

Performing this operation for all the very small- or zero-elements of the [*W*] matrix would effectively cut the circuit in different places, possibly yielding a number of smaller independent subcircuits of a quantity and size dependent on the nature of the discontinuity's geometry. It is then possible to selectively solve only the subcircuit(s) which relate to the particular mode(s) of interest, rather than the entire mode set at once. These subcircuits can be solved independently as they do not directly interact with each other.

The procedure for extracting the subcircuit containing the first mode on side A is illustrated in Fig. 2 as an example.

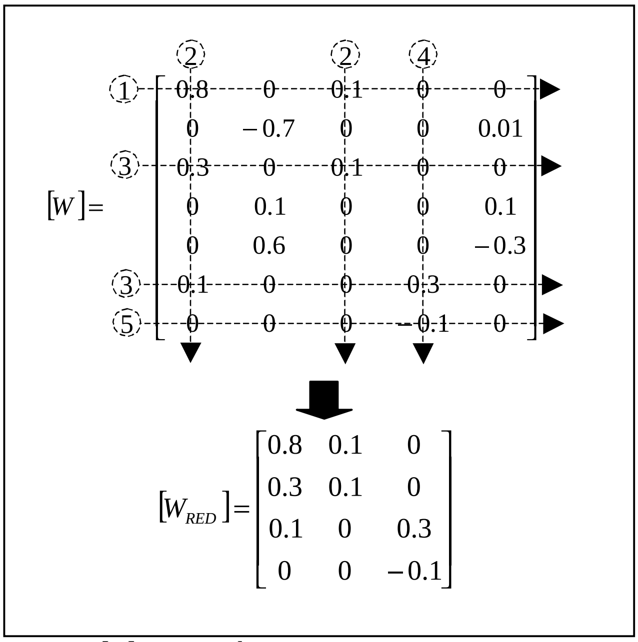

Fig. 2. [*W*] matrix reduction

First the row of the [*W*] matrix corresponding to that mode is calculated, 1. Then the columns corresponding to those row elements of magnitude greater than some threshold, e.g. 10^{-10}, are calculated, 2. Next all those rows corresponding to non-negligible entries in any of the columns are calculated, 3, and similarly through 4 and 5 and further until no further modes are added to the reduced system. The result is a reduced [*W*] matrix corresponding to a reduced set of modes on sides A and B. A similar procedure is followed to extract the subcircuit corresponding to a mode of interest on side B.

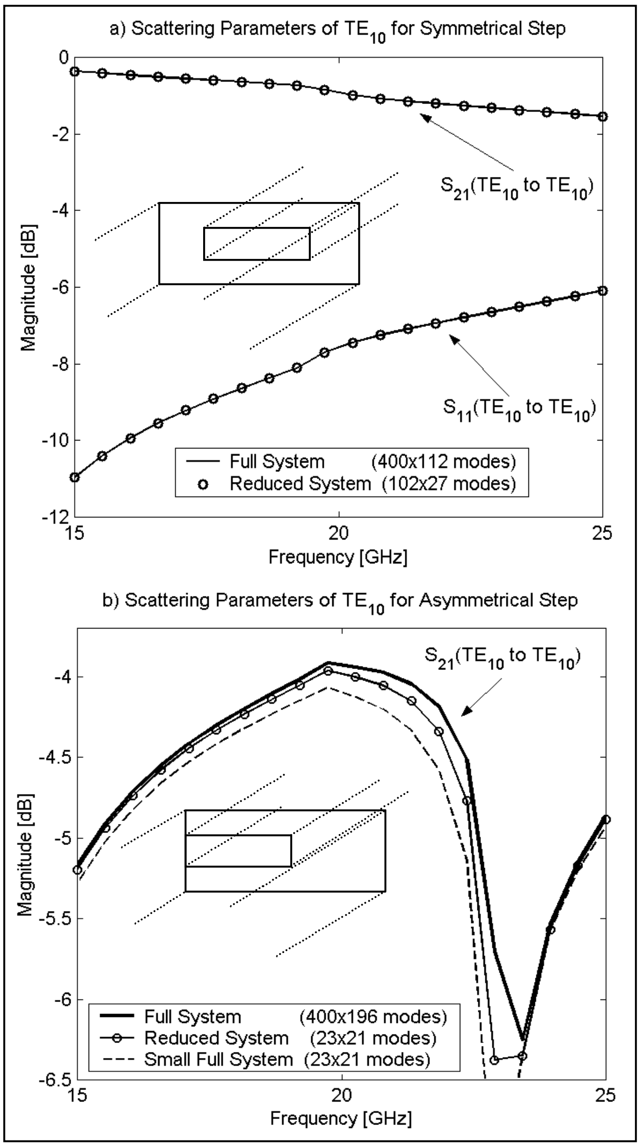

Fig 3. Reduced mode matching vs. full mode matching

It should be noted that in asymmetrical step geometries, virtually all [*W*] matrix elements have a non-zero magnitude. This implies that, to some degree, all the modes interact with each other in such a geometry. In such a case, the threshold magnitude may be set to a higher value, eliminating those transformers most approaching short and open circuits on side A and B respectively.

Fig. (3a) compares the scattering parameters for the TE_{10} mode incident on a symmetrical down-step for the standard and reduced technique. Observe that both techniques give the same result, only the reduced technique uses 102 by 27 modes to the full technique's 400 by 112 modes.

Fig. (3b) relates to an asymmetrical down-step where the [*W*] matrix element magnitude threshold has been increased to 0.1 resulting in a reduced system of 23 by 21 modes. Deviation from the full method's result is minimal, and certainly less than that of a standard analysis with 23 by 21 modes.

IV. Cascade System Analysis

If the step geometries are of a similar nature, the procedure applied to each step in a cascade system will result in the same modes being selected at each step, allowing them to be cascaded in the standard way. If, however, dissimilar steps are cascaded, which is the general case, different mode sets will be excited at adjacent steps, resulting in indirect mode interaction that must be accounted for.

A procedure for cascade system analysis has been devised that exploits the ability to split steps into a number of independent equivalent subcircuits. The procedure is outlined in Fig. 4, with calculated scattering parameters for the TE_{10} mode using both the conventional full mode technique and the new reduced mode method.

The procedure first splits all the steps into their equivalent independent subcircuits, indicated by the dots in Fig. 4. Note that simpler geometries result in a larger number of simpler subcircuits. Modes common to adjacent subcircuits are then identified. If these modes propagate sufficiently over the distance separating the adjacent steps then they can link the subcircuits, as indicated by the lines in Fig. 4.

In the example, it is desired to calculate the parameters of the TE_{10} mode, shown in dark in the figure. In this case only that part of the system in dark need be solved to accurately find it parameters. The rest of the system (in light) does not at all interact and can be neglected. Note that the second and eighth steps interact with *two* subcircuits in the third and seventh steps respectively. This is due to the dissimilar nature of the steps in the example device.

The savings in computational effort with the reduced mode system are considerable. The average equivalent step size is 13.6 modes per side in the reduced mode analysis to the 103.4 modes per side used in the full analysis. Note that, beyond dropping non-propagating modes that are attenuated to below 1% between adjacent steps, the reduced analysis is equivalent to the full analysis.

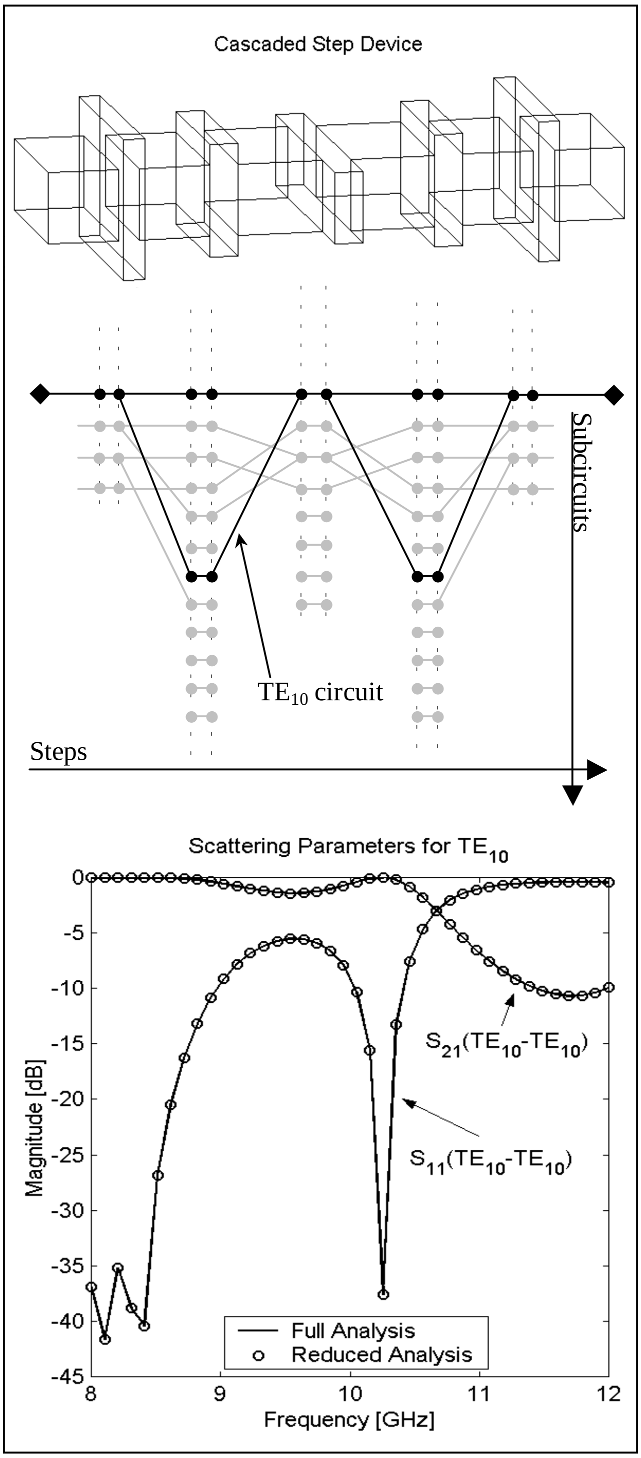

Fig 4. Cascade System Example

V. CONCLUSION

An automated technique for the intelligent selection of modes in mode matching problems has been presented. Non-interacting modes are systematically removed from the problem resulting in a smaller system to solve. The technique can also be used to speed up cascade analysis of waveguide discontinuities. Comparisons with the standard technique show exact agreement.

REFERENCES

[1] A. Wexler, "Solution of Waveguide Discontinuities by Modal Analysis," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-15, pp. 509-517, 1967.

[2] Yi-Chi-Shih, "The mode matching method," Chap. 9 of Numerical techniques for microwave and millimeter-wave passive structures, ed. by T. Itoh, John Wiley & Sons. Inc. 1989.

[3] F. Alessandri, G. Bartolucci and R. Sorrentino, "Admittance Matrix Formulation of Waveguide Discontinuity Problems: Computer-Aided Design of Branch Guide Directional Couplers," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 394-403, Feb. 1988.

[4] F. Alessandri, M. Mongiardo and R. Sorrentino, "Computer-Aided Design of Beam Forming Networks for Modern Satellite Antennas," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 1117-1127, June 1992.

[5] M. Leroy, "On the convergence of numerical results in modal analysis," *IEEE Trans. Antennas Propagat.*, vol. AP-31, pp. 655-659, July 1983.

[6] C.A.W. Vale and P. Meyer, "Modal Transforms in Rectangular Waveguide," *IEE Electronics Letters*, vol. 36 No. 16, pp. 1350-1352, August 2000.

[7] H. Jui-Pang, T. Hiraoka and H. Honma, "Equivalent network for rectangular-waveguide H-plane step discontinuity – Multi-transmission line and Multi-port ideal transformer – ," *IEEE MTT-S Intl. Symp. Digest*, vol. 2., pp. 1069-1072, Boston, 11-16 June 2000.